Indispensability Arguments and Their Quinean Heritage

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Abstract
Indispensability arguments (IA) for mathematical realism are commonly traced back to Quine. We identify two different Quinean strands in the interpretation of IA, what we label the 'logical point of view' and the 'theory-contribution' point of view. Focusing on each of the latter, we offer two minimal versions of IA. These both dispense with a number of theoretical assumptions commonly thought to be relevant to IA (most notably confirmational holism and naturalism). We then show that the attribution of both minimal arguments to Quine is controversial, and stress the extent to which this is so in both cases, in order to attain a better appreciation of the Quinean heritage of IA.

Keywords
Quine's philosophy of mathematics, indispensability arguments, Platonism, naturalism, inference to the best explanation.

Introduction and aims[*]

Word and Object (Quine 1960) contains an illuminating footnote concerning its author's views on mathematical platonism. While stressing the advantages of his 'canonical notation of quantification', and 'the restoration of law and order' that it allows in ontological

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disputes (Quine 1960: 242), Quine feels the need for the following clarification:

> A more accountable misapprehension is that I am a nominalist. I must correct it; [...] In all books and most papers I have appealed to classes and recognized them as abstract objects. I have indeed inveighed against making and imputing platonistic assumptions gratuitously, but equally against obscuring them. Where I have speculated on what can be got from a nominalistic basis, I have stressed the difficulties and limitations. (*ibidem*: 243, fn. 5)

Clearly this had not always been the case, as the complete renunciation of abstract entities declared in the first lines of Quine's and Goodman's 'Steps Towards a Constructive Nominalism' makes clear.[1] But that declaration, as Quine himself stresses, 'needs demotion to the status of a mere statement of conditions for the construction in hand' (*ibidem*). Furthermore, Mancosu 2008 has recently shown that Quine's doubts on the feasibility of that construction and the consequent complete renunciation of (mathematical) abstract entities emerged as early as in 1948.

It is now clear that Quine's stance with respect to the problem of the existence of mathematical objects is far from being a nominalist one, despite Quine's empiricist and naturalist framework. The reasons offered by Quine for this have been regimented in the form of an argument, today widely renowned as the Indispensability Argument (IA). In a nutshell, the argument appeals to the uncontroversial fact that mathematical theories are commonly and usefully applied in most of our scientific theories. Then, from the assumptions that we are justified in taking those scientific theories to be true, and that if they are true then so are the mathematical theories that we cannot avoid using in formulating them, we are led to conclude that these indispensable mathematical theories are true and that their mathematical objects exist.

Quine suggested the basic ideas underlying IA, but he offered no definite formulation of it. And from Putnam's *Philosophy of Logic* (1971) onwards, it is rare to find two formulations that completely

[1] Cf. Goodman, Quine (1947: 105): 'We do not believe in abstract entities. No one supposes that abstract entities—classes, relations, properties, etc.—exist in space-time; but we mean more than this. We renounce them altogether'.

square with each other. Nevertheless, from both *Word and Object* and a vast number of Quine's essays, it is possible to single out partly overlapping clusters of theses and notions that, in different forms, constitute the Quinean heritage of the various versions of IA available on the market.

At least two different strands in the post-Quinean discussion of IA are, according to us, identifiable. On the one hand, there is the traditional analytic attention to theories' formulation and expressive power, focusing on logico-syntactical considerations regarding the form of scientific and mathematical theories, the notion of reference to mathematical objects, and the adjudication of a proper criterion of ontological commitment. Call this *the logical point of view*. On the other hand, we find considerations stemming from the philosophy of science regarding how scientific theories work and how they are confirmed, what forms of argument are appropriate for justifying belief in those theories, and how different posited entities contribute to the overall epistemic and semantic evaluation of a given theory. Call this *the theoretical contribution point of view*. Most of the notions and theses to which many formulations of IA are currently thought to appeal – ontological commitment, indispensability, naturalism, and confirmational holism – merge aspects of both these two strands.

It is our contention that most of the available formulations of IA can be thought of as organized in a spectrum. At both ends of the spectrum lie minimal versions of the argument; minimal, that is, insofar as they feature the fewest or least controversial conceptual ingredients that are required in order to derive the desired conclusion. The arguments at each extreme are representative, respectively, of the two points of view just described. Various central shades of the spectrum are given by different versions of IA, obtained through the addition or subtraction of one or more assumptions.

It is not our present aim to review the overall structure of this spectrum. Rather, we want to show that there are both theoretical and exegetical problems in tracing both minimal versions of IA back to Quine's positions, and that these problems raise a number of concerns both regarding Quine's own way of reflecting on the issue and regarding the structure of the current debate.

From a logical point of view[2]

Colyvan's version of IA (Colyvan 2001) is a suitable representative of the sort of arguments that we would locate between the two extremes of our suggested spectrum:

[CIA] Colyvan's Indispensability Argument

i) We ought to have ontological commitment to all and only those entities that are indispensable to our best scientific theories;
ii) Mathematical entities are indispensable to our best scientific theories;

[CIA] ------------------------------

iii) We ought to have ontological commitment to mathematical entities.

According to Colyvan, the first biconditional premise 'follows from the doctrines of *naturalism* and *holism*' (Colyvan 2001: 12) – as regards, respectively, the 'only' direction and the 'all' direction of the biconditional. Let us state these two theses in a way convenient for our aims:[3]

[NAT] *Naturalism*: scientific theories are the only source of genuine knowledge. As a consequence, with respect to ontology, we are justified in acknowledging the existence *only* of those entities that are quantified over in our true or well-confirmed scientific theories.

[CH] *Confirmational Holism*: empirical evidence does not confirm scientific hypotheses in isolation, but rather scientific theories as a whole. As a consequence, with respects to ontology, we are justified in acknowledging the existence of *all* those entities that are quantified over in our true or well-confirmed scientific theories.

[2] Many of the issues in this section are explored in more details in Panza, Sereni, Fothcoming; cf. also Panza, Sereni, Unpublished.

[3] This implies radical simplifications in the formulation of these controversial theses. Naturalism, in particular, could be given milder readings. The version offered here, however, seems required in order for the 'only' direction of [CIA]'s first premise to follow from it.

Colyvan's argument is meant to support mathematical platonism – the thesis that certain mathematical objects exist – rather than what we might call *semantic realism* – the thesis that certain statements (or theories) are true, without commitment to the idea that it is objects (or anything else in particular) that make them true. Reference to ontological commitment makes this point explicit, and Colyvan is indeed assuming Quine's criterion of ontological commitment as the suitable one. Roughly speaking, and skipping some details:

> [QC] Quine's Criterion of Ontological Commitment
> The ontological commitment of a theory is given by the objects that must be counted among the values of the variables of the existentially quantified statements that are entailed by the theory.

[QC] applies to theories when they have been regimented in canonical notation and when it has been established what expressions must be indispensably employed in the reformulation thus obtained. Its application therefore requires a clear characterization of the notion of indispensability.

If we want to adhere to a Quinean formulation, as emerged at least from 'Designation and Existence' (1939) through 'On What There Is' (1948), and is clearly presented in *Word and Object*, indispensability has to be interpreted, at least partly, as a logico-syntactical feature pertaining to the formulation of a theory. Quantification (over a given sort of entity) is what is deemed indispensable when we rewrite our theories in canonical notation, if it is not possible to dispense with it by means of paraphrase and contextual definitions.

However, indispensability is a relative notion: when we want to rewrite a given theory in order to evaluate whether quantification over a given sort of entities is or isn't indispensable, we first need to establish what features of the original theory our reformulation must preserve. In other words, which specific equivalence relation allows us to say that our new theory is equivalent to the original one. Moreover, the resulting theory must *per se* enjoy a number of features that, intuitively speaking, make it a good theory. Thus a proper general clarification of the notion of (in)dispensability, restricted to quantification,[4] should take the following form:

[4] Attention to quantification is due to the attempt to evaluate the Quinean character of IA. A more general version would remain neutral as to which is the proper logico-linguistic device by which (in)dispensability is established.

> [IND] (In)dispensability: Quantification over entities X is dispensable from a scientific theory S if and only if there is a scientific theory T in which quantification over X is absent and such that:
> i) T is ε-equivalent to S, where ε-equivalence is an appropriate relation of equivalence among theories;
> ii) T is equally or more virtuous than S, given an appropriate criterion for the virtuosity of theories;
> If S quantifies over X, and there is no scientific theory T satisfying conditions (i)-(ii) above, then quantification over X is indispensable for S.[5]

Not only does this characterization show in what sense (in)dispensability is a relative, aim-specific notion (nothing is indispensable *per se*, but only relative to a specific theoretical purpose), but it also allows us to specify (and, above all, forces one to declare) in the definition of the notion, what specific theoretical feature one expects scientific theories to preserve: e.g. observational content, empirical content, expressive power, explanatory power, and so on (whether each of these is exclusive with respect to all others is a further issue). Arguably, a good candidate for ε-equivalence for what we labelled the logical point of view is 'having the same expressive power': what we are interested in is whether some mathematical vocabulary necessarily has to be employed in order to *state* certain scientific laws.

Let us come back to [CIA]. We now see that the latter is a version of IA that relies on the following four assumptions: [NAT], [CH], [QC] and an indispensability thesis based on [IND]. These are all theses or notions with Quinean origins. But do we need all this theoretical machinery in order to gain the desired conclusion? Many have stressed (Resnik 1995, Dieveny 2007, Liggins 2008) that IA can go through even without [CH] – as Colyvan himself suggests.[6] What usually goes unnoticed is that IA can go through even without [NAT]. If we try to formulate IA in a less theoretically committed way, what we get is an argument of the following form:

[5] Selection of an appropriate criterion of virtuosity might is aim-specific, and might sacrifice other features of theories commonly thought to be epistemic virtues. It is clear, for example, that Field's (1980) nominalized version of Newtonian Gravitation Theory praises ontological parsinomy and sacrifices simplicity.

[6] Cf. Colyvan (2001:37): 'As a matter of fact, the argument can be made to stand without confirmational holism: it's just that it is more secure with holism'.

[MA] *Minimal argument*

i) We are justified in believing some scientific theories to be true; 7
[We are justified in believing T to be true]

ii) Among them, some are such that some mathematical theories are indispensable to them;
[M is indispensable to T]

iii) We are justified in believing true these scientific theories only if we are justified in believing true the mathematical theories that are indispensable to them;
[We are justified in believing T to be true only if we are justified in believing M to be true]

[MA_a]------------------------------

iv) We are justified in believing true the mathematical theories indispensable to these scientific theories.
[We are justified in believing M true]

v) We are justified in believing true a mathematical theory only if we are justified in believing the objects it is about to exist;
[We are justified in believing M to be true only if we are justified in believing the objects it is about to exist]

[MA_b]------------------------------

vi) We are justified in believing the objects which the indispensable mathematical theories are about to exist.
[We are justified in believing the objects M is about to exist]

The argument, if sound, establishes for given mathematical theories what Field (1982:501) calls *theoretical indispensability*. It claims that (we are justified in believing that) mathematical theories are true (or mathematical objects exist) on the grounds of considerations about the proper formulation or expressive power of theories. Such an argument – relying, among the aforementioned theses, only on

[7] [MA] is stated in epistemic terms. It speaks of our justification in believing certain theories to be true and certain objects to exist – as is the case in Colyvan's argument and others. Notice that also a non-epistemic version of the two arguments can be offered by reformulating steps (*i*), (*iii*), (*iv*) and (*v*) in such a way that justification is not mentioned in them. This difference hinges upon two different conceptions of ontology, as respectively either a descriptive or a normative discipline. In the end, this leaves us with four different (minimal, though for different reasons) versions of IA: [MA_a], [MA_b], and the non-epistemic versions of both.

[QC] and an indispensability thesis based on [IND] – shows several advantages over Colyvan's.

One is that it shows that two different conclusions might be reached by different versions of IA: [MA_a] is an argument for semantic realism, from which it is possible (but not mandatory) to get to platonism if one adds premise (*v*), which in fact can be taken to express (a generalizes version of) [QC].

Secondly, notice that what is needed in order to justify premise (*i*) is some form of scientific realism, and this is a weaker position than [NAT]. Scientific realism sees scientific theories as *a* genuine source of knowledge, but need not consider them as *the only* genuine source of knowledge. If IA is made to appeal to both [CH] *and* [NAT], it provides sufficient *and* necessary conditions for its conclusion(s). Thus neither the semantic realist nor the platonist conclusion cannot be reached for all those theories (and their objects) that do not find application in true or well-confirmed scientific theories. Quine accepted this conclusion[8], but a more plausible version of IA might want to avoid it, as does [MA]. [MA] is not by itself inconsistent, for example, with the belief that we can gain mathematical knowledge (about mathematical statements or mathematical objects) through *a priori* arguments.

Scientific realism is something Quine clearly championed[9], but it seems that the whole complex of Quine's theses overdetermines a proper version of IA: some version will follow if both [CH] and [NAT] are assumed, but they need not be assumed in order for *any* version of IA to be offered. That [CH] and [NAT] are not indispensable to IA was already implicit in Putnam's formulation in *Philosophy of Logic* (Putnam 1971:347):

> So far I have been developing an argument for realism roughly along the following lines: quantification over mathematical entities is indispensable for science, both formal and physical, therefore we should accept such quantification; but this commits us to accepting the existence of the mathematical entities in question.

[8] With varying attitudes: cf. Quine (1986:400) and Quine (1995:56-57). Maddy (1992) has argued against this consequence of a Quinean version of IA (cf. also Parsons 1978, Maddy 1992, Leng 2002, Colyvan 2007).

[9] Issues with indeterminacy and ontological relativity apart: cf. Putnam (1988).

No mention is made here of naturalism nor holism.[10.] If we follow Putnam, we can still have a proper version of IA and yet consider the part of the Quinean heritage consisting of [NAT] and [CH] as dispensable from the argument – unless held for independent reasons and for these reasons alone made to bear on IA.

The theory-contribution point of view

Let us move to the other extreme of our suggested spectrum, and focus on another form of reasoning that is often appealed to in connection with IA. We can take our clue from a passage of Quine's often suggested to be a statement of IA:

> Ordinary interpreted scientific discourse is as irredeemably committed to abstract objects – to nations, species, numbers, functions, sets – as it is to apples and other bodies. All these things figure as values of the variables in our overall system of the world. The numbers and functions contribute just as genuinely to physical theory as do hypothetical particles. (Quine 1981: 149-50)

We take Quine's argument to be the following: if it can be argued that mathematical entities contribute to scientific theories in a relevantly similar way to how theoretical entities contribute to those theories, then there is (either in the positive or in the negative sense) as much reason to believe that the latter exist as there is reason to believe that the former exist.

Even though [CH] might have been a working hypothesis of Quine's throughout his works, there is no explicit mention of it in the quotation above. Colyvan (2001) and Baker (2009) have accordingly suggested a reading of IA based on the notion of theory contribution that is independent of [CH] (Colyvan explicitly takes this formulation to be of Quinean heritage). This is obtained by stressing that IA seems intimately connected with arguments for scientific realism about theoretical entities. Theory contribution is seen in both cases as evidence for existence, and mathematical entities are thus argued for by means of an inference to the best explanation (IBE). The generic

[10] As concerns naturalism and the 'only' direction in [CIA], Putnam recently stressed this point again. Cf Putnam Forthcoming.

form of IBE can be expressed as follows (where T is a theory and X is a set of data):

i) X
ii) T, T_2, …, T_n are explanations of X
iii) T explains X better than $T_2 \ldots T_n$
iv) We ought rationally to believe the theory that best explains X
[IBE]-------------------------
v) We ought rationally to believe T

Scientific realists argue that if the postulation of theoretical entities contribute towards a given explanation being best, we have reason to believe that they exist. If one believes, as Quine apparently does, that mathematical entities contribute to the (explanatory) goodness of scientific theories in just the same way that theoretical entities do, then we ought to, by considerations of consistency, be realists about mathematical entities too (Colyvan 2006). We might easily state a version of IA that argues for mathematical platonism by means of [IBE] (call it IA_{IBE}).

It is, of course, not enough to simply postulate that mathematical entities contribute to scientific theories in the same way as theoretical entities, and much work has been done in recent literature that tries to argue that mathematical entities do contribute in this way (e.g Colyvan 2008; Baker 2005, 2009). Rather than add to that debate, we shall argue: (i) that IA_{IBE} is different from other versions of IA both in kind and in content, and (ii) that IA_{IBE} does not have the kind of Quinean heritage that Colyvan attributes to it.

It is important to notice that [IBE] is an ampliative mode of inference. We have seen that [CIA] involves four of the ingredients introduced above: [IND], [NAT], [CH] and [QC]. To what extent are any of these presupposed in the ampliative variety of IA? On an appropriate understanding of "best theories", only one of them is.

Consider first [NAT]. IA_{IBE} presupposes only scientific realism, and it was already pointed out that scientific realism is a weaker position than naturalism.

Now consider [CH]. Appeal to theory contribution is supposed to make [CH] redundant: if mathematical theory M contributes appropriately to a scientific theory T that counts as a best explainer, we thereby have a justification for M, and there is thus no need to adopt [CH].[11]

The notion of indispensability therefore ceases to play any role. It is implicit that mathematics is indispensable to our current best theories, because any part of our current best theories (that ontologically commits to some kind of entity) is considered indispensable to that theory. This is partly what it means for a theory to be "best". If rival empirically adequate theories existed, that had less ontological commitments, those theories would be considered best, everything else being equal.

How about [QC]? In order to get from the truth of any theory whatsoever to a claim about what exists something like [QC] will be needed. As all that IA_{IBE} can establish is that we have reason to believe that some theory is true, some variety of [QC] will be needed for inferring that mathematical entities (or any other entities mentioned in those theories) exist. So out of all of the ingredients above, only [QC] (or something similar) is doing any work in IA_{IBE}.

It is not at all clear what the Quinean heritage of IA_{IBE} would be, since only one of the traditional Quinean-inspired premises will be doing any work in it. Colyvan clearly understands [CIA] as Quinean in spirit (Colyvan 2001). However, the argument from theory contribution that we find in the writings of Quine is very different in nature from the way that scientific realists think about [IBE] and theory contribution as they are employed in that mode of inference.

[IBE] is argued to be a reliable mode of inference by scientific realists, in virtue of best explanations being truth tracking. As argued by van Fraassen (1980), one could easily interpret scientists as choosing some theory over others because it is more *useful* to employ certain theories over others, e.g. because certain theories are easier to work with in virtue of their simplicity or the like. This latter criterion is pragmatically motivated in that it reflects our interests and what we find useful: it is thus not to be related to issues of truth at all. For those scientific realists who argue for realism by use of [IBE] (who are those who would potentially be persuaded by IA_{IBE}) it is imperative that a pragmatic reading of [IBE] is ruled out. Rather surprisingly,

[11] [CH] could be used to argue for scientific realism, but it is actually at odds with current varieties of scientific realism, since these hold that only parts of our scientific theories are confirmed by empirical evidence. See Folina (1999).

however, Quine understood theoretical virtues in a way that is incompatible with the required realist understanding of [IBE].

Quine's argument for mathematical platonism by way of theory contribution.

Since there is no direct evidence to support the hypothesis that theoretical entities exist, Quine sensibly suggests that we need to look for what might count as 'indirect' evidence for their existence. A look at scientific practice shows that everything else being equal, simple theories are judged to be better theories than complex theories:

> The molecular physicist is, like us, concerned with commonplace reality, and merely finds that he can simplify his laws by positing an esoteric supplement to the esoteric universe (…) No matter if physics makes molecules or other insensible particles seem more fundamental than the objects of common sense, the particles are postulated for the sake of simple physics. (Quine 1966: 236-241)

Quine makes two observations here. The first is that in one sense scientists are, like 'us' ordinary people, concerned with commonplace reality. The second is that scientists postulate the existence of entities like molecules for the sake of simplifying laws. Thus far, this latter claim looks rather ontologically innocent, amounting only to a description of how physicists practice physics, and it is ambiguous between a realist and an antirealist account of theoretical entities. When we turn to Quine's view on how we come to form beliefs about objects of common-sense reality the above observations become significant:

> If we have evidence for the existence of the bodies of common sense, we have it only in the way in which we may be said to have evidence for the existence of molecules. The positing of either sort of body is good science insofar as it helps us formulate our laws. (Quine 1966: 237)

All of the evidence that we consider as relevant to the existence of visible objects is in fact evidence in the same sense of 'evidence' relevant to the positing of molecules. Furthermore, we have here an indication that Quine actually thinks that positing bodies makes for 'simpler' theories, in the sense that doing so is 'helpful'. In other places, he is absolutely clear that the sense of evidence he has in mind is best construed as pragmatically motivated:

> Actually I expect that tables and sheep are, in the last analysis, on much the same footing as molecules and electrons. Even these have a continuing right to a place in our conceptual scheme only by virtue of their indirect contribution to the overall simplicity of our linguistic or conceptual organization of experience; for note that even tables and sheep are not direct sensations… It would be senseless to speak of a motive for this archaic and unconscious posit [common-sense bodies], but we can significantly speak of its function and its survival value; and in these respects the hypothesis of common sense external objects is quite like that of molecules and electrons. (Quine 1953: 210).

Quine has now suggested a number of things: if we have evidence for the existence of objects of our commonplace reality we only have evidence in the same sense that we have evidence for the existence of molecules; objects of commonplace reality are theoretic in the same ways as theoretical entities like molecules because they are given within a conceptual scheme; the reasons we have for positing the existence of molecules are that doing so is 'helpful' and 'useful' for the purpose of physics, it has a 'function' and a 'survival value'.

According to Quine there is no standard of reality outside of the standards given in our conceptual scheme, or theory, of the world: 'Everything to which we concede existence is a posit from the standpoint of a description of the theory building process, and simultaneously real from the standpoint of the theory that is being built'. (Quine 1960: 22)

We cannot significantly question the reality of the external world, or deny that there is evidence of external objects in the testimony of our senses; for, to do so is simply to dissociate the terms "reality" and "evidence" from the very applications which originally did most to invest those terms with whatever intelligibility they may have for us. (Quine 1957: 2)

Because we take the kind of evidence that we have for making existence claims about objects – 'their assumption helps [man] organize experience' – to be defining of what we mean by evidence, we should by consideration of consistency also hold that this kind of evidence is sufficient for making claims about unobservables. Quine then argues that since pragmatic value is sufficient for making claims about observables, pragmatic value is also sufficient for making claims about unobservables: 'The benefits of the molecular doctrine which so impressed us [earlier], and the manifest benefits of the aboriginal posit of ordinary bodies, are the best evidence of reality we can ask'. (1966: 238-239)

With this rather permissive view of evidence, we can now revisit the issue of whether mathematical entities and theoretical entities are evidentially on a par. When we look to at least some of our scientific theories it is no doubt true that mathematics contributes towards the formulation of theories that are simpler (or the like) than theories formulated without the use of mathematics. Thus, by Quine's characterisation of evidence, mathematics contributes to theories in a way that warrants belief that mathematical objects quantified over in mathematical theories exist:

> I think the positivists were mistaken (...). Existence statements (...) do admit of evidence, in the sense that we can have reasons, and essentially scientific reasons, for including numbers or classes or the like in the range of the values of our variables. Numbers and classes are favoured by the power and facility they contribute to theoretical physics and other systematic discourse about nature. (Quine 1969: 97-98)

So quite independently of consideration about [CH], Quine produced an argument for believing that mathematical entities exist (a similar reconstruction of Quine can be found in Chihara 2004). Quine's argument works by first pointing out the parity of evidence for believing that ordinary sized objects – posited in our commonsense 'theory' of the world exist, and for believing that molecules – posited in some of our scientific theories about the world – exist. Then it is pointed out that the evidential grounds we have for believing that molecules exist are similar to those for believing that mathematical entities exist. In each case, posits are postulated because of pragmatic and purpose-oriented reasons.

Concluding remarks

Our discussion pointed to two different directions along which the widespread claim that most current versions of IA are, in some way or other, faithful to Quine's original ideas should be qualified. This result also emerges from the consideration that the most discussed version of IA at present, i.e. [CIA], is, despite its superficial simplicity (or maybe just for that reason), ambiguous between two different kinds of argument, one deductive and one ampliative.

On the one hand, as regards our discussion of what we labelled the 'logical point of view' on IA, it turns out that by sticking to notions and theses that Quine respectively employed and endorsed, versions of IA can be formulated that are far less committing than most available versions. Much of the recent debate has focused on whether it is possible both to avoid problems posed by the alleged assumptions of confirmational holism and naturalism of IA, and to formulate IA without these assumptions. But it seems clear – as apparently seemed clear also to Putnam – that this formulation can be obtained, and those problems avoided, without being unfaithful to Quine's thought: it is just a matter of disregarding those theoretical ingredients that overdetermine a valid version of IA. Discussion of holism and naturalism will thus be of relevance to IA only in so far as proponents of IA independently support either thesis. By themselves, they are irrelevant to the question of whether the sought-for realist or platonist conclusion can be obtained by a version of IA that is Quinean in its essential traits.

On the other hand, our discussion of 'theory contribution' has made it clear that Quine's understanding of theory contribution and evidence, is incompatible with current scientific realist strategies for defending scientific realism realism by means of [IBE] as they understand it. Thus one cannot base a reading of Quine, according to which Quine endorses [IBE] as it is understood by current scientific realists, in the writings of Quine himself. One might well formulate Quine's argument as an argument with the same formal structure as an [IBE] argument, but the notion of 'best explanation' should then be defined in terms of pragmatic value. Of course, within Quine's framework this is of no consequence, since one can construct an argument for mathematical platonism by parity considerations on the basis of what Quine says. But the way in which Quine understands the idea of theory contribution is not in line with how current realists understand it.

All these issues would deserve further inquiry, and we submit that progress can be made in the understanding of the historical and philosophical import of IA once the argument's Quinean heritage is brought into sharper focus.

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